Minimum-Weight Design of an Orthotropic Shear Panel with Fixed Flutter Speed

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Abstract

THIS paper deals with the weight minimization of rectangular flat panels placed in a high supersonic flowfield and subject to a flutter speed constraint. In the establishing of the structural operator a pure transverse shear plate model was used, which may be considered as a complement of the Love-Kirchhoff type model. By using the theory of optimal control of distributed parameter systems, necessary conditions for the minimum-weight panel are derived. These are supplemented with a condition ensuring that the flutter speed of the optimal panel coincide with the prescribed one. It is shown that the optimal thickness distribution is symmetrical with respect to the panel midpoint. Numerical rough estimates obtained via Galerkin's method are presented.

Contents

The field of weight minimization of panels subjected to aeroelastic constraints has been investigated thoroughly during the past decade, as it may be inferred from the specialized literature. Throughout these investigations, whether dealing with one-dimensional (see Refs. 1-4) or two-dimensional aeroelastic optimization problems,[5-8] the appropriate structural operator was established on the basis of the Love-Kirchhoff type model. As it is known, this model involves the ab-initio disregard of transverse shear effects. In contrast to this approach, a somewhat opposite structural type model is used here, in which the rigidities in transverse shear are considered as finite, and in bending as negligible (such a panel will be termed a pure transverse shear panel). This model—first introduced by Armand[9]—is practically motivated by the advent of new composite materials that enjoy exotic properties. A generalized form of this structural model, including transverse shear orthotropicity effects, will be used here for approaching the present aeroelastic optimization problem.

The structure to be analyzed consists of an elastic, rectangular flat thin panel ($a \times b$) of nonuniform thickness $h = h(x_1, x_2)$, where Ox_1x_2 denotes the in-plane coordinate system (Ox_1 is the streamwise coordinate, while Ox_2—the spanwise one—coincides with the panel leading edge). The panel is exposed to a high supersonic gas flow over its upper face.

The aeroelastic optimization problem dealt with here consists of finding the thickness distribution which minimizes the panel weight, while maintaining the same flutter speed as that of a uniform-thickness reference panel. As usual,[8] the reference panel is defined as the panel of uniform thickness h_0 having the same orthotropy characteristics and boundary conditions as its counterpart of nonuniform thickness.

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In deriving the governing equations it will be assumed that the freestream Mach number is sufficiently high so as to consider linear piston theory aerodynamics as valid; structural and aerodynamic damping parameters are not retained in the analysis. In this framework, the aeroelastic equilibrium equation is given by

$$\bar{G}_{13}\frac{\partial}{\partial x_1}\left(h\frac{\partial \hat{w}}{\partial x_1}\right)+\bar{G}_{23}\frac{\partial}{\partial x_2}\left(h\frac{\partial \hat{w}}{\partial x_2}\right) - \frac{\kappa p_\infty M_\infty}{G_{\rm ref}}\frac{\partial \hat{w}}{\partial x_1} - \frac{\rho h}{G_{\rm ref}}\frac{\partial^2 \hat{w}}{\partial \tau^2} = 0 \tag{1}$$

where $\hat{w} = \hat{w}(x_1, x_2, \tau)$ denotes the transverse deflection of the panel, $\bar{G}_{i3} \equiv G_{i3}/G_{\rm ref}$ $(i = 1,2)$ are the nondimensional elastic moduli in transverse shear in the $i3$ planes ($G_{\rm ref}$ denotes a conveniently chosen shear modulus, while direction 3 coincides with the upnormal to the Ox_1x_2 plane), ρ mass density of the panel, κ the polytropic gas coefficient, p_∞ and M_∞ the undisturbed pressure and Mach number, and τ the time.

Defining the dimensionless variables $\bar{x}_1 = x_1/a$, $\bar{x}_2 = x_2/b$, $\hat{\bar{w}} = \hat{w}/a$ and assuming simple harmonic motions $\hat{\bar{w}}(x_1, x_2, \tau) = \bar{w}(x_1, x_2)\exp(j\omega\tau)$, the governing equation becomes

$$\bar{G}_{13}\frac{\partial}{\partial x_1}\left(t\frac{\partial w}{\partial x_1}\right)+\Phi^2\bar{G}_{23}\frac{\partial}{\partial x_2}\left(t\frac{\partial w}{\partial x_2}\right) - \Lambda\frac{\partial w}{\partial x_1} + \bar{\omega}^2 t w = 0 \tag{2}$$

In Eq. (2) and in the following the overbars affecting x_1, x_2, and w are dropped; in addition, $t(x_1, x_2) \equiv h(x_1, x_2)/h_0$ denotes the nondimensional thickness distribution; h_0 the uniform panel thickness; $\Lambda \equiv a\kappa p_\infty M_\infty/(G_{\rm ref}h_0)$ the velocity parameter; $\bar{\omega}^2 \equiv \pi^2(\omega^2/\omega_0^2)$, $1/\omega_0^2 \equiv \rho a^2/(\pi^2 G_{\rm ref})$ the frequency parameter; and $\Phi \equiv a/b$ the inverse aspect ratio.

The aeroelastic optimization problem will be stated as follows. Find the optimal thickness ratio distribution $t(x_1, x_2)$ that minimizes the performance index

$$J = \int_0^1\int_0^1 t(x_1, x_2)\,\mathrm{d}x_1\mathrm{d}x_2 \tag{3}$$

subject to the partial differential constraint Eq. (2) with adequate boundary conditions, where the speed parameter Λ is held fixed during the optimization process and equal to the critical flutter parameter of the reference panel $(\Lambda_0)_*$.

By extending the theory of optimal control of distributed parameter systems as exposed in Ref. 9 to the present problem (see, also, Ref. 8), the necessary optimality conditions reduce to the following Euler-Lagrange and control equations, respectively,

$$\bar{G}_{13}\frac{\partial}{\partial x_1}\left(t\frac{\partial \mu_3}{\partial x_1}\right)+\Phi^2\bar{G}_{23}\frac{\partial}{\partial x_2}\left(t\frac{\partial \mu_3}{\partial x_2}\right) + (\Lambda_0)_*\frac{\partial \mu_3}{\partial x_1} + \bar{\omega}^2 t\mu_3 = 0 \tag{4}$$

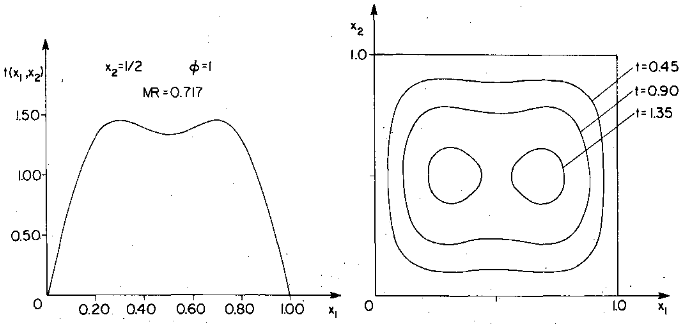

Fig. 1 Thickness distribution of minimum-weight orthotropic shear panel with fixed flutter speed.

$$\bar{G}_{23} + \bar{G}_{13}\frac{\partial w}{\partial x_1}\frac{\partial \mu_3}{\partial x_1} + \Phi^2 \bar{G}_{23}\frac{\partial w}{\partial x_2}\frac{\partial \mu_3}{\partial x_2} - \bar{\omega}^2 w \mu_3 = 0 \qquad (5)$$

which, together with the governing Eq. (2), constitute a system of three simultaneous differential equations involving the three dependent variables $w(x_1,x_2)$, $\mu_3(x_1,x_2)$, and $t(x_1,x_2)$, subject to the boundary conditions $w=0$, $\mu_3=0$ along $x_1=0$; 1, $x_2=0$; 1 (which are appropriate for simply-supported (SS) edges). In Eqs. (4) and (5) μ_3 stands for a Lagrange multiplier.

As may be remarked, Eqs. (2) and (4) differ only by the sign of the odd-derivative term, which may be interpreted physically as a reversal of the flow direction. Equation (4) is referred to as the adjoint to the governing Eq. (2). In light of the boundary conditions, it may be concluded that the eigenvalues of the two equations are identical.

At this point, it should be remarked that the frequency parameter $\bar{\omega}^2$ [which plays the role of an eigenvalue in Eqs. (2) and (4)] is yet undetermined. As is known, in the absence of damping, the flutter critical parameters of the aeroelastic system (i.e., velocity and frequency) may be obtained in the $(\bar{\omega}^2, \Lambda)$ plane as the coordinates of the point where two of the eigenfrequencies are coalescing.[14] Accordingly, the value of $\bar{\omega}^2$ which is intervening in the optimal solution must be determined so as to constitute, together with the velocity parameter $(\Lambda)=(\Lambda_0)_*$ ab initio held fixed, the flutter parameters of the minimum-weight panel. Having in view the adjoint character of Eqs. (2) and (4), the aforementioned requirement may be expressed in terms of Plaut's condition,[10] which in our case reads

$$\int_0^1 \int_0^1 t w \mu_3 \, \mathrm{d}x_1 \mathrm{d}x_2 = 0 \qquad (6)$$

Thus, the present aeroelastic optimization problem reduces finally to Eqs. (2) and (4-6) in the unknowns t, w, μ_3, $\bar{\omega}^2$ and subject to appropriate boundary conditions. The flutter critical parameter $(\Lambda_0)_*$ and $(\bar{\omega}_0^2)_*$ of the pure transverse shear panel of uniform thickness can be determined from the set of flutter equations

$$(\bar{\omega}^2 - \bar{\omega}_{mn}^2) C_{mn} - 2\Lambda \sum_{s=1,2,\ldots} \left(\frac{2s+1-m}{2s+1} C_{2s+1-m,n} + \frac{m-2s+1}{2s-1} C_{m-2s+1} - \frac{2s-1+m}{2s-1} C_{2s-1+m,n} \right) = 0, \quad (m,n=1,2,\ldots) \qquad (7)$$

in which the frequency-coalescence condition may be used.[14] Equation (7) is obtained by applying the Galerkin procedure in Eq. (2) (specialized for $t=1$) in which w is represented —consistently with SS-edge conditions—by

$$w(x_1,x_2,\tau) = \exp(j\omega\tau) \sum_{m=1,2,\ldots} C_{mn} \sin m\pi x_1 \sin n\pi x_2$$

In Eq. (7) $\bar{\omega}_{mn}^2 = \pi^2(\omega_{mn}^2/\omega_0^2)$ denotes the nondimensional eigenfrequencies of the SS uniform panel, where $\omega_{mn}^2 = \pi^2\omega_0^2(\bar{G}_{13}m^2 + \bar{G}_{23}n^2\Phi^2)$.

In the optimality Eqs. (2) and (4-6) by changing x_i into $1-x_i$ $(i=1,2)$, it may be shown in this framework as well (see also Refs. 11, 12, and 8) that a possible solution of the problem could be expressed as $\mu_3(x_1,x_2) = Bw(1-x_1, 1-x_2)$, $w(x_1,x_2) = \mu_3(1-x_1, 1-x_2)/B$, and $t(x_1,x_2) = t(1-x_1, 1-x_2)$. The last expression shows that the optimal thickness distribution is symmetrical with respect to the panel midpoint, where B will be set equal to -1 (see Ref. 13). The Lagrange multiplier μ_3 being thus eliminated, the optimal problem reduces to the two nonlinear partial differential equations, Eqs. (2) and (5), the last one rewritten in terms of t, w, and $\tilde{w} \equiv w(1-x_1, 1-x_2)$ as

$$1 - \frac{\bar{G}_{13}}{\bar{G}_{23}}\frac{\partial w}{\partial x_1}\frac{\partial \tilde{w}}{\partial x_1} - \Phi^2 \frac{\partial w}{\partial x_2}\frac{\partial \tilde{w}}{\partial x_2} + \frac{1}{\bar{G}_{23}}\bar{\omega}^2 w \tilde{w} = 0 \qquad (8)$$

to which the flutter instability condition (6) (also expressed in terms of t, w, and $\tilde{w}$) is to be adjoined and used for determining the critical flutter frequency $(\bar{\omega}_{\mathrm{opt}}^2)_* \equiv \pi^2(\omega^2/\omega_0^2)_{\mathrm{opt}}^*$ of the minimum-weight panel.

For an SS square panel $(\Phi=1)$, Galerkin's technique[7] leads to the following rough estimate of the optimal thickness distribution

$$t(x_1,x_2) = (1.660 \sin\pi x_1 + 0.3298 \sin 3\pi x_1)\sin\pi x_2$$

which is independent of the orthotropicity ratio (see full paper) and yields a weight saving of 28%. This distribution is depicted in Fig. 1 and shows a trend similar to previous evaluations based on the Love-Kirchhoff type model.[4-7,13]

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